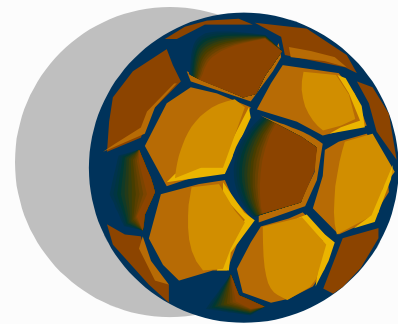


The Math of Kicking

Ödön Bléidit Master subtitle style
www.bullbots.org





The Goal

- Understand the math and physics behind kicking the soccer ball a distance of 20 feet.
- Keep the ball in the playing field. (No penalties!)
- Stay within the physical constraints of the robot.



The Constants

- The size and weight of the ball
 - 9" diameter
 - 16 ounces (1 lb)
- The height of the playing field walls
 - 42"
- Acceleration due to gravity
 - 32ft/sec² or 9.8m/sec²



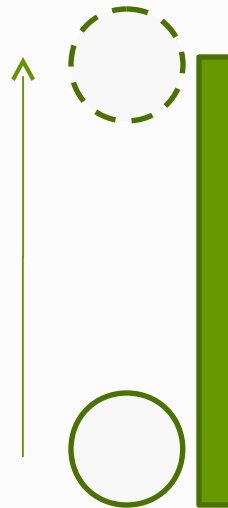
The Equations

- Distance, acceleration and time:
 $D = \frac{1}{2} AT^2$
- Velocity, acceleration and time:
 $V = AT$
- Distance velocity and time:
 $D = VT$
- Force, mass and acceleration:
 $F = MA$



Staying on the playing field

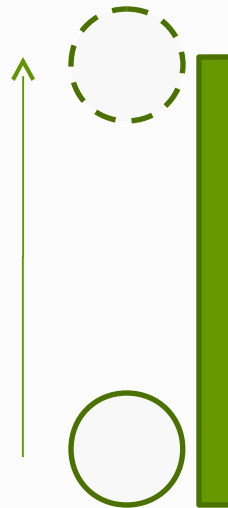
- Wall height is 42"
- Ball diameter is 9"
- Therefore, the ball should not be kicked higher than ???





Staying on the playing field

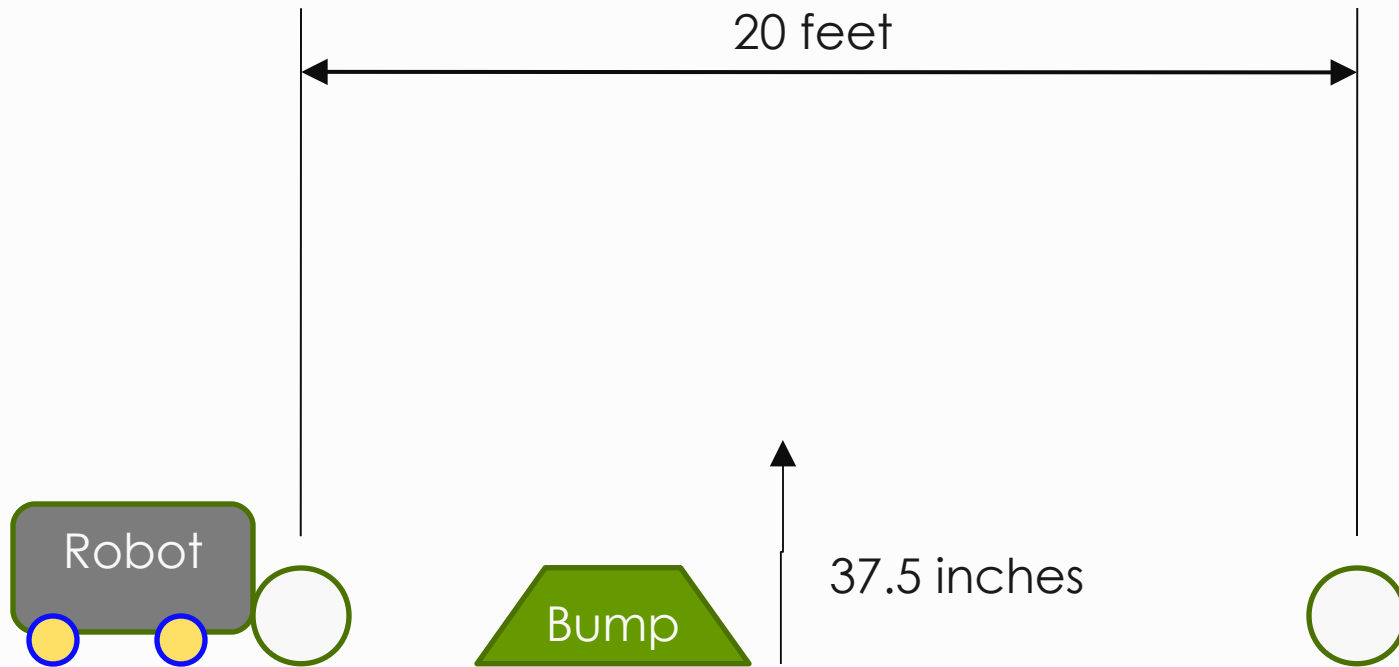
- Wall height is 42"
- Ball diameter is 9"
- Therefore, the ball should not be kicked higher than $42'' - 4.5'' = 37.5''$





The trajectory

- Let's kick this ball with style!





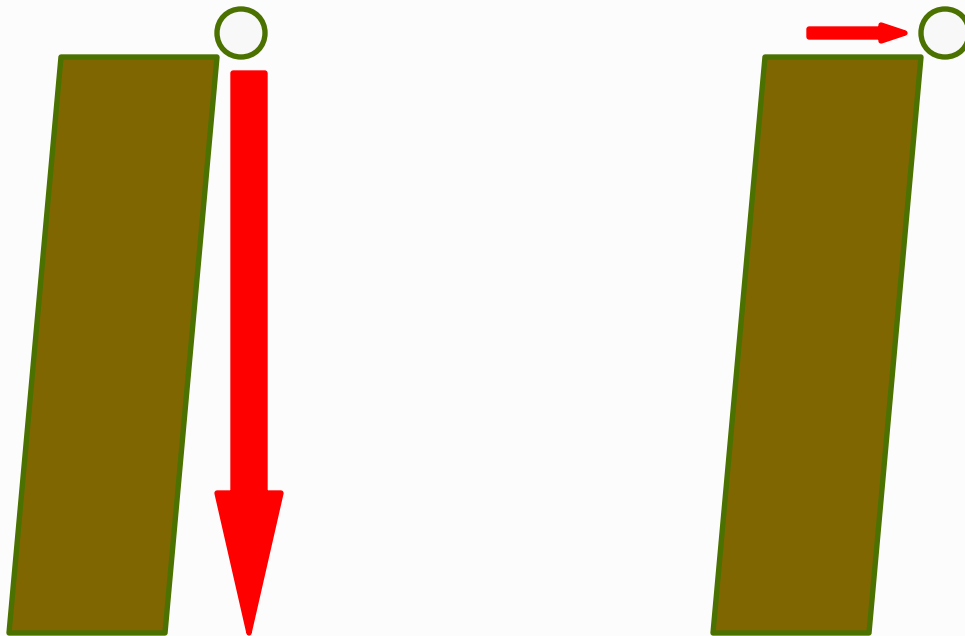
Major Concept to be Understood!

- The **horizontal** motion of an object can be separated from the **vertical** motion of the object.
- The vertical motion is affected by gravity, and is independent of the weight of the object.
- The horizontal motion is governed by its inertia and surface friction.



Example:

An object that is pushed off a cliff or horizontally kicked off a cliff will hit the ground at the same time.



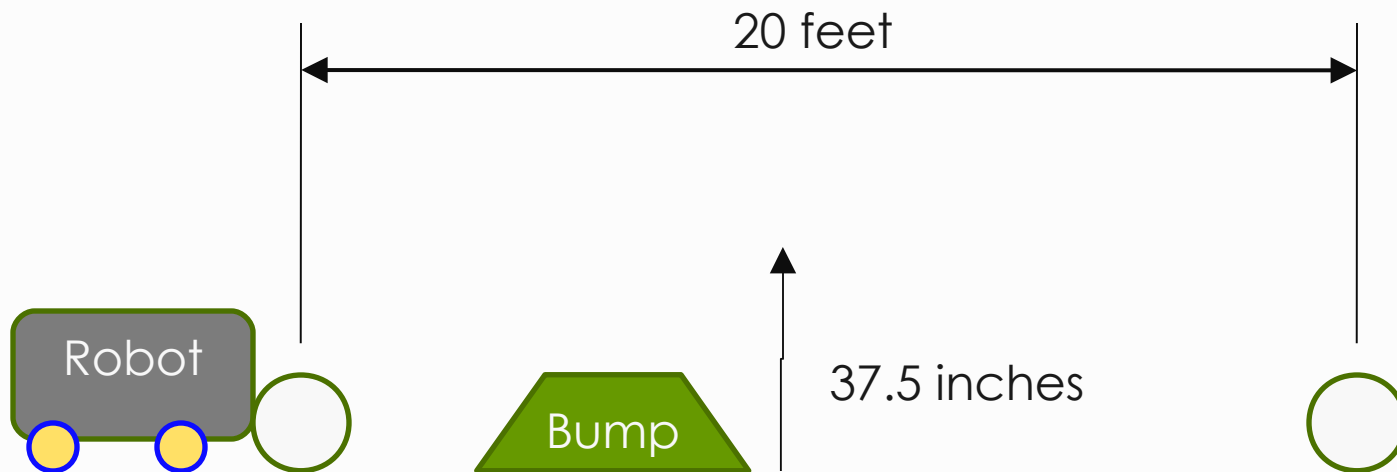


Step 1: Figure out how long the ball is in the air

This is ALL about the vertical element and is bounded by the 33 inch vertical maximum.

$D = 1/2 AT^2$, where $D = 37.5''$ (3.125') and $A = 32 \text{ft/sec}^2$

$3.125 = 1/2 \cdot 32 T^2$ Therefore: $T = \sqrt{3.125/16} = 0.44 \text{ sec}$





Step 1: Continued

- What goes up must come down!
- So the time the ball is in the air is:

$$2T = 2 * 0.44 = 0.88 \text{ seconds}$$

Try it!



Step 2: How fast does the ball go vertically?

- From the time of the kick to the time the ball reaches the top of the wall is 0.44 seconds.
- Gravitational acceleration is 32 ft/sec²
- Apply the formula:

$$V=AT = 32\text{ft/sec}^2 * 0.44\text{sec} = 14.1 \text{ ft/sec}$$



Step 3: **How fast does the ball go horizontally?**

- The ball is in the air for 0.88 seconds.
- In that time, it must travel 20 feet.
- Neglecting air resistance, the balls speed is constant.
- Use the formula:

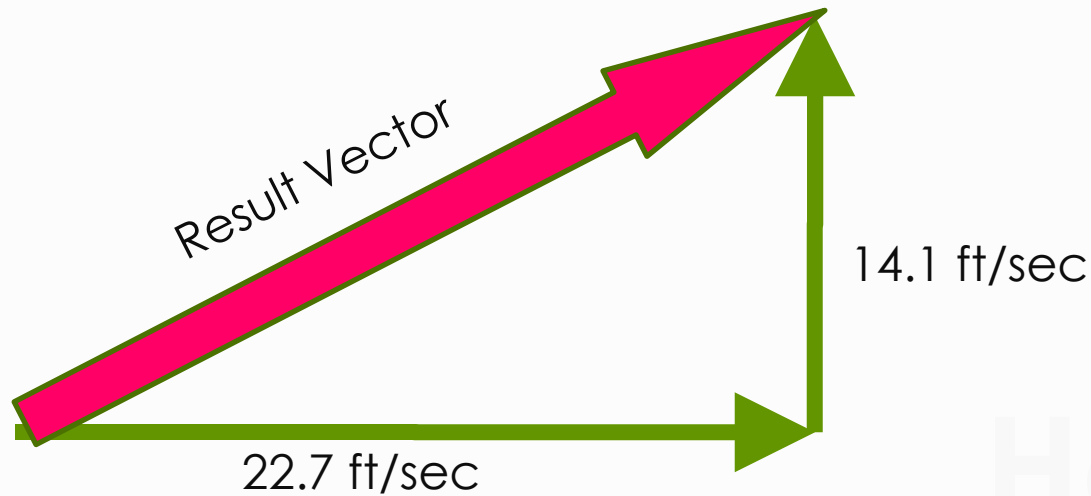
$$D = VT, \text{ or } D/T = V = 20 \text{ feet}/0.88 \text{ seconds}$$

$$V = 22.7 \text{ ft/sec}$$



Step 4: Compute the actual velocity of the ball

- We need to add the vertical velocity and the horizontal velocity – **in vectors!**

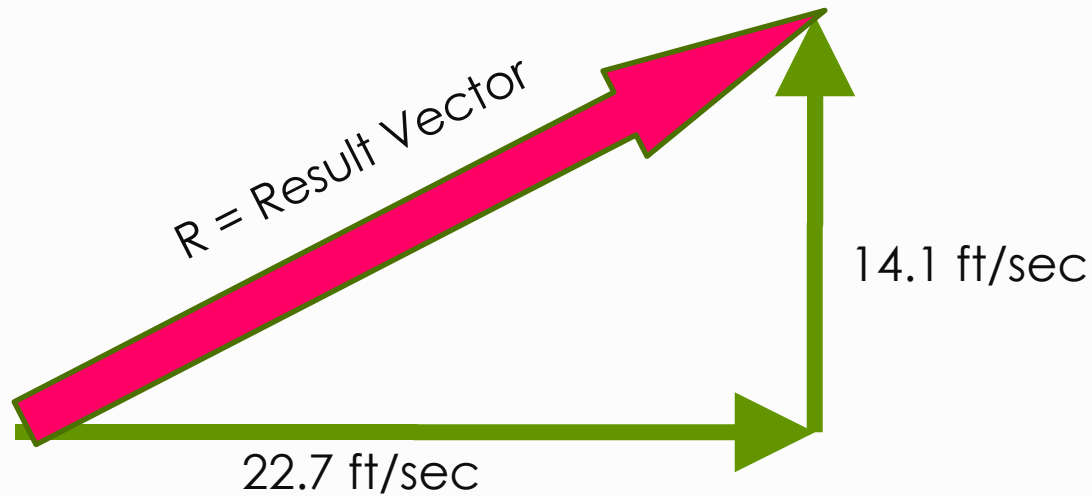


How?



Step 4: Compute the actual velocity of the ball

- To add these vectors requires the Pythagorean Theorem: $R^2 = H^2 + V^2$

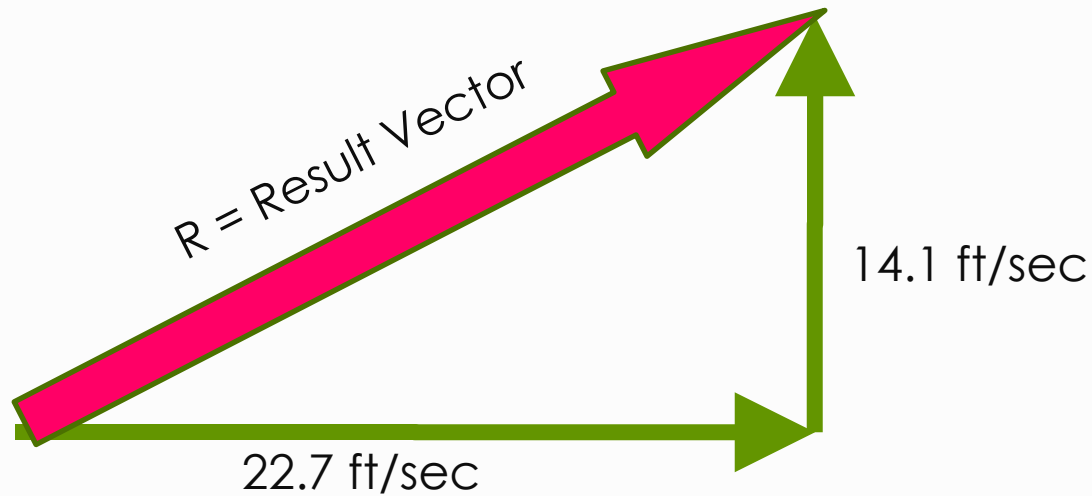




Step 4: Compute the actual velocity of the ball

$$R^2 = H^2 + V^2, \text{ or } R = \sqrt{H^2 + V^2}$$

$$R = \sqrt{(22.7^2 + 14.1^2)} = 26.7 \text{ ft/sec}$$

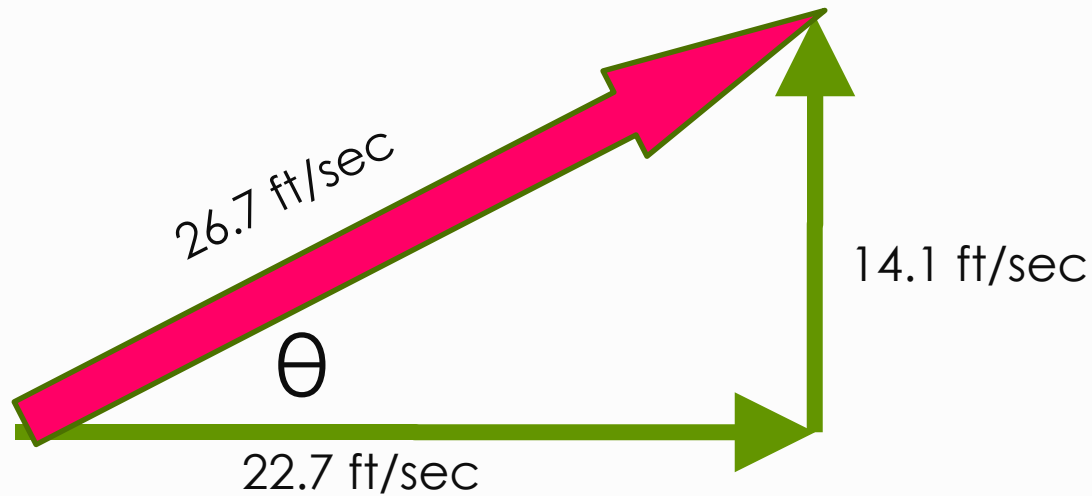




Step 5: Compute the launch angle of the ball

$$\theta = \text{Arctan} (14.1/22.7)$$

$$\theta = 32 \text{ degrees}$$





Step 6: Compute the kick

- Rule R19: The ball can extend in under the robot frame a maximum of 3".

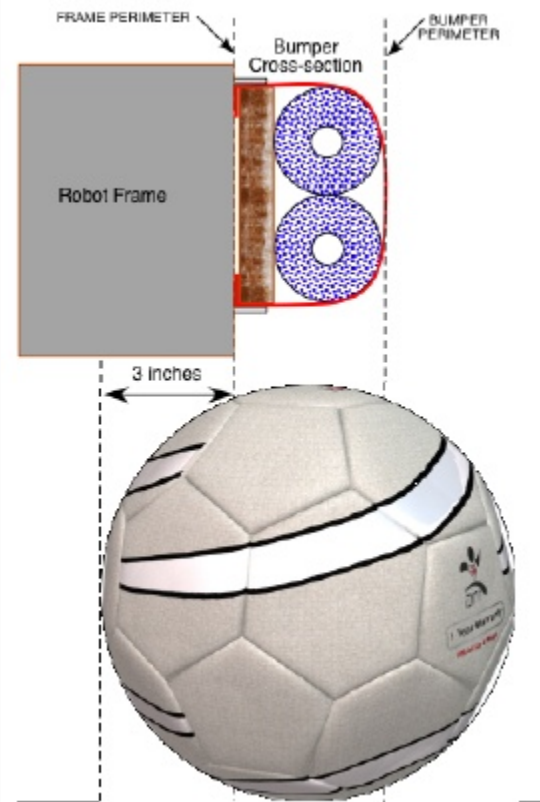


Figure 8-5



Step 6: Continued

- Rule G30: The Kicker can extend out to the outside of the bumper.
- Therefore: We have a distance of about 6" to deliver the crushing blow...

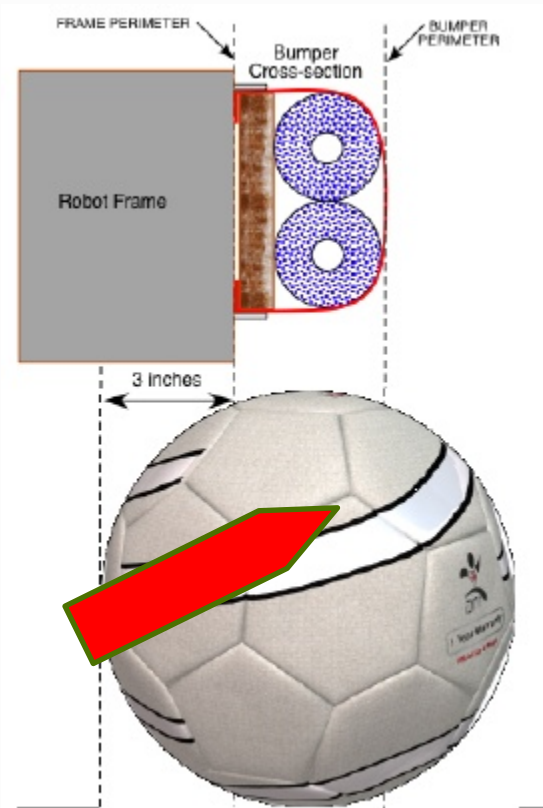


Figure 8-5



Step 6: Continued

- We have a distance of 6" (0.5 feet) to accelerate the ball to its launch velocity of 26.7 ft/sec.
- Two useful equations:
$$V = AT \text{ and } D = \frac{1}{2} AT^2$$



Step 6: Continued

- $V = 26.7 \text{ ft/sec} = AT$
- $D = 0.5' = \frac{1}{2} A * T^2$
- Substitute and solve:
- $A = V^2/2D = 26.7^2 / 2 * 0.5 = 713 \text{ ft/sec}^2$



Step 7: The force of the kick

- The equation: $F = MA$
- The ball has a weight of 1 lb, and a mass of $1\text{lb}/32\text{ft}/\text{sec}^2$
- $A = 713 \text{ ft}/\text{sec}^2$
- Therefore: $F = 1/32 * 713 = \mathbf{22.3 \text{ lbs}}$



The End!
Or the beginning...